4 Quadratic Factorization and Equations

Basic Practice

1. Factorize each of the following.
(a) $w^2 + 9w + 14$
(b) $x^2 - 11x + 24$
(c) $y^2 + 6y - 16$
(d) $z^2 - 3z - 10$
(e) $a^2 + 40 + 13a$
(f) $6b + b^2 - 27$

2. Factorize each of the following.
(a) $2p^2 + 11p + 14$
(b) $5q^2 - 23q + 12$
(c) $4r^2 + 4r - 35$
(d) $6s^2 - 7s - 5$
(e) $3x^2 + 3 - 10x$
(f) $11x + 6x^2 + 4$

3. Factorize each of the following.
(a) $2y^2 + 12y + 16$
(b) $4m^2 - 20m + 24$
(c) $5n^2 + 10n - 40$
(d) $6n^2 - 26n - 20$
(e) $28c + 8c^2 - 60$
(f) $-15 - 39p + 18p^2$
(g) $3m^2 - 12mn + 12n^2$

4. **(a)** Factorize $2x^2 + x - 1$.
(b) Hence, evaluate $2 \times 99^2 + 99 - 1$.

5. The general term of a sequence is $T_n = pn^2 + n - 3$, where p is a constant. The 3rd term of the sequence is 18.
(a) Find the value of p.
(b) Hence, express the general term as a product of two factors in n.
(c) Use the answer in **(b)** to find the 101st term.

9. **(a)** Make y the subject of the formula $z = \dfrac{x - 2y}{3y}$.

(b) Hence, find the value of y when $x = 22$ and $z = -\dfrac{5}{2}$.

10. **(a)** Simplify $\dfrac{1}{x} - \dfrac{1}{y}$.

(b) Hence, find the value of $\dfrac{1}{x} - \dfrac{1}{y}$ if $6xy = -1$ and $6(x - y) = 5$.

11. **(a)** Factorize

(i) $x^2 + 3x - 10$,

(ii) $2x^2 - 4x$.

(b) Simplify $\dfrac{x^2 + 3x - 10}{2x^2 - 4x}$.

(c) **(i)** Make x the subject of the formula $y = \dfrac{x^2 + 3x - 10}{2x^2 - 4x}$.

(ii) Hence, find the value of x when $y = 13$.

Further Practice

12. Simplify each of the following algebraic fractions.

(a) $\dfrac{3x - 6}{x^2 - 3x + 2}$

(b) $\dfrac{-6x + 18}{x^2 + x - 12}$

(c) $\dfrac{10x - 50}{x^2 - 25}$

(d) $\dfrac{2x + 6}{8x^2 - 72}$

(e) $\dfrac{3x^2 + 7x - 6}{3x^2 - 20x + 12}$

(f) $\dfrac{4x^2 - 11x - 20}{-5x^2 + 18x + 8}$

(g) $\dfrac{8x^2y - 3x^3}{9x^2 - 64y^2}$

(h) $\dfrac{5x - 6y + 30xz - 36yz}{2 - 72z^2}$

(i) $\dfrac{4x^2 - 20xw + 25w^2}{4xy - 14xz - 10wy + 35wz}$

13. Simplify each of the following.

(a) $\dfrac{2x - 3y}{4ab - 16} \times \dfrac{ab^2 - 4b}{6xz - 9yz}$

(b) $\dfrac{x^3 - 4x}{3x - 6} \times \dfrac{9x}{x + 2}$

(c) $\dfrac{25x^2 - y^2}{x^2 - 4xy} \times \dfrac{3x^3}{15x + 3y}$

(d) $\dfrac{x^2 + 2x + 1}{x^2 - 3x - 4} \times \dfrac{x^2 - 16}{2x^2 - 2x - 4}$

(e) $\dfrac{3xz - zy - 3wx + wy}{9x^2 - 12xy + 4y^2} \times \dfrac{2y - 3x}{w - z}$

(f) $\dfrac{6x - 2x^2}{3y + 4} \div \dfrac{9 - x^2}{4w + 3wy}$

(g) $\dfrac{x^2 - 5x + 6}{x^2 - 4} \div \dfrac{x^2 - 6x + 9}{4x - 12}$

(h) $\dfrac{4x^2 + 4xy + y^2}{9x^2} \div \dfrac{5y^2 - 20x^2}{3y - 6x}$

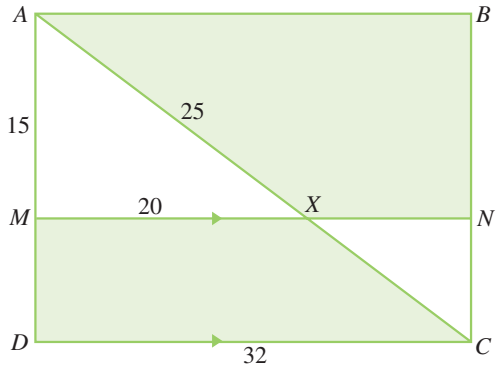
(i) $\dfrac{4x^2z - 16z}{xy^2 + 2y^2} \div \dfrac{(4z)^2}{(2y)^3}$

28. Under an enlargement with center of enlargement at (2, 4), $\triangle ABC$ with vertices $A(1, 3)$, $B(3, 2)$, and $C(5, 2)$ is mapped onto $\triangle PQR$.
If P is a point in the y-axis,
(a) state the coordinates of P and find the scale factor of the enlargement,
(b) find the coordinates of Q and R,
(c) show that the area of $\triangle PQR$ is 4 times the area of $\triangle ABC$.

Challenging Practice

29. **(a)** Using a graph with x ranging from –8 to 6 and y from –10 to 6 and a scale of 1 cm to represent 1 unit on both axes, draw and label $\triangle ABC$ with vertices $A(6, -1)$, $B(6, -4)$, and $C(3, -2)$.
(b) **(i)** Reflect $\triangle ABC$ in the line $y = x$ and label the image as $\triangle A_1B_1C_1$.
(ii) Write down the coordinates of A_1, B_1, and C_1.
(c) **(i)** Translate $\triangle A_1B_1C_1$ by –4 units in the x-direction and –4 units in the y-direction and label the image as $\triangle A_2B_2C_2$.
(ii) Write down the coordinates of A_2, B_2, and C_2.
(d) **(i)** Rotate $\triangle A_2B_2C_2$ through 180° about the fixed point (–3, –4) and label the image as $\triangle A_3B_3C_3$.
(ii) Write down the coordinates of A_3, B_3, and C_3.
(e) **(i)** Rotate $\triangle A_3B_3C_3$ through 90° clockwise about the fixed point (2, –5) and label the image as $\triangle A_4B_4C_4$.
(ii) Write down the coordinates of A_4, B_4, and C_4.
(f) Describe a single the transformation that will map $\triangle A_1B_1C_1$ directly $\triangle A_3B_3C_3$.

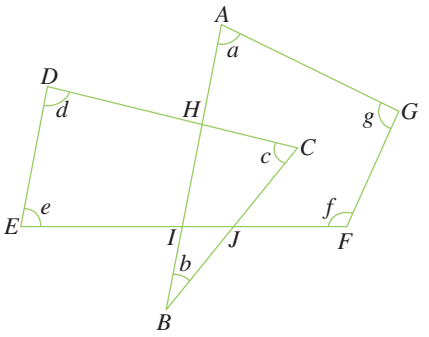
30. A rectangular piece of paper, $ABCD$, is cut into 4 smaller pieces along the lines AC and MN such that $\triangle AMX$ is similar to $\triangle ADC$. $AX = 25$ cm, $AM = 15$ cm, $MX = 20$ cm, $DC = 32$ cm, and MN is parallel to DC.



(a) Calculate the length of
(i) NX,
(ii) CN,
(iii) CX.
(b) Hence, find the area of
(i) $ABNX$,
(ii) $CDMX$.

Enrichment

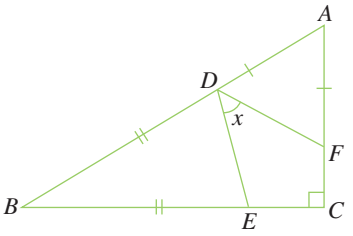
27.



(a) In the figure, $AHIB$, BJC, CHD, and $EIJF$ are straight lines.
Find $m\angle a + m\angle b + m\angle c + m\angle d + m\angle e + m\angle f + m\angle g$.

(b) If the sum of the interior angle of a regular polygon is the same as the sum in **(a)**, find the number of sides of the polygon.

28. In the figure, $\triangle ABC$ is a right-angled triangle with $m\angle ACB = 90°$. D, E, and F are points on the sides of $\triangle ABC$ such that $AD = AF$ and $BD = BE$.

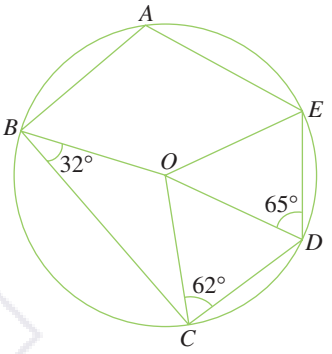


(a) Find the value of x.

(b) If an exterior angle of a regular polygon is equal to $\angle x$, find the number of sides of the polygon.

(c) If $m\angle CAB = 64°$, find $m\angle BED$.

29. In the figure, O is the centre of the circle. $m\angle OBC = 32°$, $m\angle OCD = 62°$, and $m\angle DOE = 65°$.



Find

(a) $m\angle BOC$,

(b) $m\angle BOE$,

(c) $m\angle BAE$.