Part 1: Ratio and Fraction — 6 sessions

Objectives

- Compare quantities using ratios.
- Express a ratio in its simplest form.
- Express a ratio as a fraction of a quantity.
- Express a fraction of a quantity as a ratio.
- Solve word problems which involve ratios, using pictorial models.

Materials

- *Primary Mathematics 5A* textbook and Teacher's Guide
- Connect-a-cubes or other cubes
- Counters

Homework

- Workbook Exercise 8
- Workbook Exercise 9
- Workbook Exercise10
- Workbook Exercise 11

Notes

In *Primary Mathematics 5A*, students learned to write ratios which involved two or three quantities. The also learned to find the simplest form of a ratio. If your students did not use *Primary Mathematics 5A*, you may want to do unit 5 of *Primary Mathematics 5A* along with what is covered here for pp. 21-23 of the *6A* textbook.

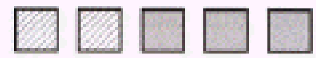
A ratio is a comparison of the relative size of two or more quantities. In a ratio, quantities can be compared without specifying the unit, as long as the same unit is used for both quantities. *In measurement, a unit is required.* A rope that is 6 *feet* long is not the same length as one that is 6 *meters* long. However, one rope can be accurately compared to another rope without specifying the unit. For example, one rope can be specified as twice as long as the other. The ratio of their lengths is then 2 : 1 whether the ropes are measured in feet, meters, inches or centimeters.

Equivalent ratios are ratios where the relative sizes of the quantities remain the same, but the unit is different. 200 : 300, 20 : 30, 10 : 15, and 2 : 3 are equivalent ratios. Equivalent ratios can be found by multiplying or dividing each term by the same number. If the terms of a ratio have a common factor, we can then simplify the ratio by dividing each term by the common factor. If there is no such common factor, the ratio is already in its *simplest form*. 2 : 3 is a ratio in its simplest form.

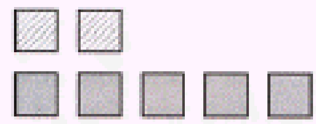
The importance of the concept of "unit" is constantly emphasized in *Primary Mathematics*. A unit can be a "one" or "one collection of several items". If the unit is 1 vegetable, the ratio of 200 carrots to 300 onions is 200 : 300. But if the unit is 100 vegetables, the ratio of 200 carrots to 300

Activity 3.1c **Ratio and fractions**

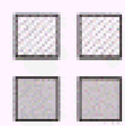
1. Illustrate the relationship between ratios and fractions.
   - Use cubes, blocks, or draw pictures on the board. Display 2 blocks of one color, such as yellow, and 3 of another, such as red.



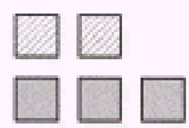
     - Ask students to express the number of yellow blocks as a fraction of the total number of blocks. The number of yellow blocks is $\frac{2}{5}$ of the total number of blocks.
     - Add two more red blocks, and move the yellow ones to above the (now 5) red blocks. Ask students to express the yellow blocks as a fraction of the total number of blocks. The number of yellow blocks is $\frac{2}{7}$ of the total number of blocks.



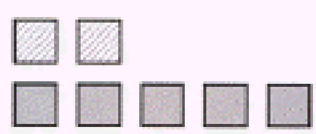
     - Now ask students to express the number of yellow blocks as a fraction of the number of red blocks. The number of yellow blocks is $\frac{2}{5}$ of the number of red blocks.
     - Point out that the denominator (bottom) of the fraction now refers to the number of red blocks (5) instead of all the blocks (which is now 7). We are finding yellow blocks as a fraction of red blocks, not as a fraction of total blocks.
     - Ask students for the ratio of yellow blocks to red blocks (2 : 5). Since the number of yellow blocks is $\frac{2}{5}$ the number of red blocks, we can say that the ratio of yellow blocks to red blocks is 2 : 5.
   - Display 2 yellow and 2 red blocks.



     - Ask students to compare the red blocks to the yellow blocks. There are as many red as yellow blocks, so the ratio of the number of yellow blocks to the number of red blocks is 1 : 1.
     - Now add a red block and ask students to compare the *red* blocks to the *yellow* blocks. There are one and a half times as many red as yellow blocks. We can say that the number of red blocks is $\frac{3}{2}$, or $1\frac{1}{2}$, of the number of yellow blocks. There are $\frac{3}{2}$ as many red blocks as yellow blocks, and the ratio of the number of red blocks to the number of yellow blocks is 3 : 2.



     - Add 2 more red blocks. Ask students student to express the number of *red* blocks as a fraction of the number *of yellow* blocks. The number of red blocks is $\frac{5}{2}$ of the number of yellow blocks.



     - Now ask for the ratio of red to yellow blocks (5 : 2). If we are told that the ratio of red blocks to yellow blocks is 5 : 2, then we can also say that the number of red blocks is $\frac{5}{2}$ *of the number of yellow blocks.*
     - The number of yellow blocks is $\frac{2}{5}$ *of the number of red blocks.*

Activity 3.2a **Proportion**

1. Discuss **textbook pp. 30-31.**
   - Point out that when we increase the number of buckets of cement, the number of buckets of sand should also be increased by the same factor.
   - The amounts of cement and of sand are kept in the same *proportion*.

$\dfrac{\text{Number of buckets of cement}}{\text{Number of buckets of sand}}$ is always equivalent to $\frac{5}{3}$ when expressed in its simplest form.

   - Ask students to solve these proportions by finding equivalent fractions.
   - Lead students to see that a proportion can be described by using ratios (top of p. 31).
   - Discuss other examples which involve proportions. For example:
   - ➢ $\dfrac{\text{weight on the moon}}{\text{weight on earth}} = \dfrac{1}{6}$

     Have students find what their weight would be if they were standing on the moon.
   - ➢ Discuss the scale on a map, and have students estimate some actual distances between cities or other features. Have them use the map and a ruler. Emphasize that they are applying the concept of proportions.

2. Discuss **tasks 1-3, textbook pp. 31-32.**
   - Students should see that we can solve a problem that involves proportion as a ratio problem.
   - In task 2(b), we are given the value of 1 unit, and can find the value of 3 units using multiplication.
   - In task 2(c), we are given the value of 3 units, and can find the value of 1 unit using division.
   - In task 3, have students first show the value for 1 unit, and then find the answer to the problem.

3. (a) 3 units = 12 ℓ

   1 unit = $\frac{12}{3}\ \ell = 4\ \ell$

   2 units = $4\ \ell \times 2 = 8\ \ell$

   (b) 5 units = 10 ℓ

   1 unit = $\frac{10}{5}\ \ell = 2\ \ell$

   3 units = $2\ \ell \times 3 = 6\ \ell$

Workbook Exercise 12

Activity 3.2b **Practice**

1. Have students work on the problems in **Practice 3B, textbook p. 33**, and share their solutions. All these problems can be solved by first finding the value for 1 unit, as shown here for problem 9:

total units = 4 units + 5 units + 6 units = 15 units

15 units = 60 cm

1 unit = $\frac{60}{15}$ = 4 cm

4 units = 4 cm × 4 = 16 cm

The shortest side is 16 cm.